

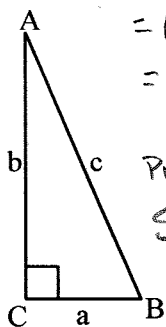
Tues 3/17

Pre Calculus

Trig Identities Review #1

Name Key

1. Prove: $\sin^2 A + \cos^2 A = 1$



$$\begin{aligned} &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 \\ &= \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} \end{aligned}$$

Pyth. Thm states $a^2 + b^2 = c^2$.
So $\frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1 \checkmark$

2. Given: $\sin^2 \theta + \cos^2 \theta = 1$

Prove: $1 + \cot^2 \theta = \csc^2 \theta$

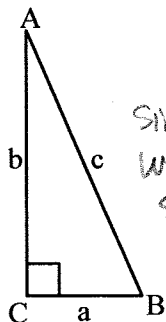
$$\begin{aligned} \frac{1}{\sin^2 \theta} (\sin^2 \theta + \cos^2 \theta) &= (1) \frac{1}{\sin^2 \theta} \\ \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} &= \frac{1}{\sin^2 \theta} \\ 1 + \cot^2 \theta &= \csc^2 \theta \checkmark \end{aligned}$$

3. In each example: (a) tell whether or not the expression is true; (b) If false, state correctly

Two ways to correct

- $-\sin^2 \theta = \cos^2 \theta - 1$
- $\sin^2 \theta = \cos^2 \theta - 1$ false; $\sin^2 \theta = 1 - \cos^2 \theta$
 - $\tan^2 \theta - \sec^2 \theta = -1$ True
 - $\csc^2 \theta - \cot^2 \theta = 1$ True

5. Prove: $\sec^2 B - \tan^2 B = 1$



$$\begin{aligned} &\left(\frac{c}{a}\right)^2 - \left(\frac{b}{a}\right)^2 \\ &\frac{c^2}{a^2} - \frac{b^2}{a^2} = \frac{c^2 - b^2}{a^2} \end{aligned}$$

Since Pyth. Thm says $a^2 + b^2 = c^2$,
We know $c^2 - b^2 = a^2$,
So $\frac{c^2 - b^2}{a^2} = \frac{a^2}{a^2} = 1 \checkmark$

6. Given: $\sin^2 \theta + \cos^2 \theta = 1$

Prove: $1 + \tan^2 \theta = \sec^2 \theta$

$$\begin{aligned} \frac{1}{\cos^2 \theta} (\sin^2 \theta + \cos^2 \theta) &= (1) \frac{1}{\cos^2 \theta} \\ \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} &= \frac{1}{\cos^2 \theta} \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \tan^2 \theta &= \sec^2 \theta \checkmark \end{aligned}$$

7. In each example: (a) tell whether or not the expression is true; (b) If false, state correctly

- $\sin^2 \theta = 1 - \cos^2 \theta$ True
- $1 + \sec^2 \theta = \tan^2 \theta$ False $\sec^2 \theta - 1 = \tan^2 \theta$
or $1 + \tan^2 \theta = \sec^2 \theta$
- $1 - \csc^2 \theta = \cot^2 \theta$ False $1 - \csc^2 \theta = -\cot^2 \theta$
or $\csc^2 \theta - 1 = \cot^2 \theta$

8. Simplify: $(1 + \tan^2 \theta) \cot^2 \theta$

$$\begin{aligned} &= \sec^2 \theta \cdot \cot^2 \theta \\ &= \frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \frac{1}{\sin^2 \theta} \\ &= \boxed{\csc^2 \theta} \end{aligned}$$

9. Prove: $1 + \cot^2 A = \csc^2 A$

$$\begin{aligned} 1 + \left(\frac{b}{a}\right)^2 &= \frac{a^2}{a^2} + \frac{b^2}{a^2} = \frac{a^2 + b^2}{a^2} \end{aligned}$$

Pyth. Thm says $a^2 + b^2 = c^2$,
So, $\frac{a^2 + b^2}{a^2} = \frac{c^2}{a^2} = \left(\frac{c}{a}\right)^2 = \csc^2 A \checkmark$

